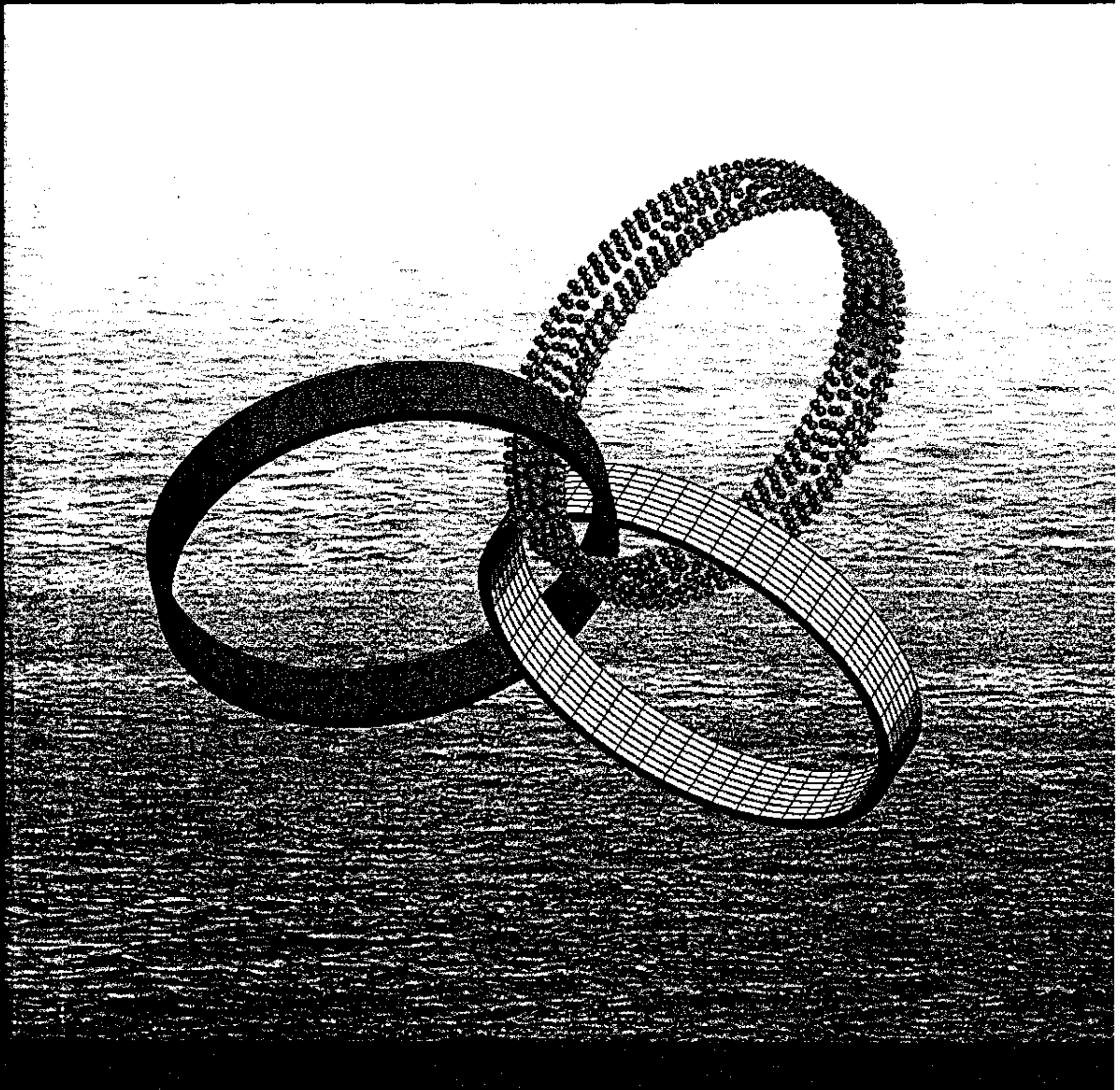
Computational Methods for Coupled Problems in Science and Engineering

Edited by:
M. Papadrakakis, E. Oñate and B. Schrefler



Computational Methods for Coupled Problems in Science and Engineering

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PREFACE

This volume contains the Abstracts and de CD-Rom Proceedings of the papers presented at COUPLED PROBLEMS 2005, "First International Conference on Computational Methods for Coupled Problems in Science and Engineering", held in Santorini Island, Greece from May 25-27, 2005.

The increasing necessity to solve complex problems in science and engineering accounting for all the coupling occurring on the different scales of the problem requires the development of new ideas and methods which can effectively provide a new level of mathematical modeling and numerical solution. This will lead to a deeper understanding of physical phenomena and a greater improvement in engineering design.

The objective of COUPLED PROBLEMS 2005 is to present and discuss state of the art mathematical models, numerical methods and computational techniques for solving accurately and with affordable computing times coupled problems of multidisciplinary character in science and engineering. Emphasis will be given on showing the potential of new computational methods for solving practical multidisciplinary problems of industrial interest.

The conference goal is to make a step forward in the formulation and solution of real life problems with a multidisciplinary vision accounting for all the complex couplings involved in the physical description of the problem.

This is the first International Conference on this subject organized in the framework of Thematic Conferences of the European Community on Computational Methods in Applied Sciences (ECCOMAS).

The conference is jointly organized by the Greek Association for Computational Mechanics (GRACM), the National Technical University Athens (NTUA), and the International Center for Numerical Methods in Engineering (CIMNE), in co-operation with the University of Padova and Universitat Politècnica de Catalunya (UPC). The organizers, as well as the Technical Advisory Panel, acknowledge the encouragement and support of ECCOMAS and the International Association for Computational Mechanics (IACM) under whose auspices this conference is held.

Altogether about 180 lectures will be given, including nine plenary lectures, which reflect the current state of the research and advances in engineering practice in this field.

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Session Organizers
We would like to thank the Session Organizers for their help in the setting up of the Technical Programme of the Conference: *Carlos Agelet de Saracibar, Daniele Boffi, Michele Chiumenti, Eduard Divo, Lucia Gastaldi, Manfred Kaltenbacher, George Karniadakis, Alain Kassab, Reinhard Lerch, Wing Kam Liu, Eugenio Oñate, Jacques Périaux, Jean-Philippe Ponthot, Ernest Rank, C. T. Sun and Wolfang Wall.*

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Conjugate Heat Transfer and Thermoelasticity

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High-Order Algorithms for Multifield Problems
Minisymposium organized by Ernest Rank and George Karniadakis

Multiscale Simulation of Coupled Physical Problems
Minisymposium organized by Wing Kam Liu and C. T. Sun

CONTRIBUTED SESSIONS

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Coupling Algorithms

FEM - BEM Coupling

Fluid - Structure Interaction

Heat and Mass Transport

Multifield - Multicoupled Problems

Simulation Coupling

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International Center
for Numerical Methods in Engineering

Generalitat de Catalunya

Int. Conf. on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2005
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A NUMERICAL PROCEDURE FOR STRUCTURE LIFE ASSESSMENT

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Key words: Stress Intensity Factors (SIFs), J-Equivalent Domain Integral Method (J-EDI), eXtended Finite Element method (X-FEM), Crack Growth, Life Assessment.

Abstract. *Numerical methods, especially the finite element (FE) method, have been widely used in computational fracture mechanics. However, modelling of the crack and its growth in the traditional FE framework require that FE mesh coincidences with the internal boundary of the crack and desire some technique for remeshing. In the PAK software that is developed on the Faculty of Mechanical Engineering of the University of Kragujevac, beside traditional FE method and X-FEM (eXtended Finite Element Method) is incorporated. The X-FEM is recently developed technique for modelling cracking within the finite element (FE) framework that use meshes independent of the crack configuration and thus avoid remeshing. In the X-FEM a discontinuous function and asymptotic crack-tip displacement fields are added to the finite element approximation to account for the crack using the notion of partition of unity (PU). This enables the domain to be modelled by finite elements with no explicit meshing of the crack. Numerical integration for the enriched elements, linear dependence and the corresponding solution techniques for the system of equations, as well as the accuracy of the crack tip fields are addressed. For calculation stress intensity factors (SIFs) we used J-integral. In this paper equivalent domain integral (EDI) method for evaluation of the J-integral is presented. The developed numerical model for J-EDI method is incorporated in the PAK software. The J-EDI method for determination SIFs in the traditional FE and X-FEM framework is used.*

This method applied to a number of test cases. Numerical results are compared with relevant theoretical values. Using the developed software, the stress intensity factors of the steam turbine housing were calculated and compared with the corresponding results obtained with COSMOS software. The results indicate that the developed procedure can be very useful tool for modelling real structures containing cracks. Also, at the final part of this paper, the example of the crack growth simulated by using remeshing free X-FEM (FE framework) is presented and obtained numerical results are compared with available data from referenced literature.

Application of the J-EDI integral is suitable for applications because it relies on use of the domain integrals rather than contour integrals. Obtained numerical results show a small influence of the choice of the J-integral domain integration on value of the stress intensity factor. Analysis of the complex 3-D problems shows that a stable crack growth is predicted in nominal regime of the analyzed structure, while 2-D analysis shows a rapid increase of the stress intensity factor for the large crack depth.

1 INTRODUCTION

Studies of the fracture mechanics emerged in the early twentieth century. Among a number of researchers, Griffith's idea of "minimum potential energy" provided a foundation for all later successful theoretical studies of fracture, especially for brittle materials. But it was not until after World War II that fracture mechanics developed as a discipline. Derived from Griffith's theorem, the concept of energy release rate G, was first introduced by Irwin, in a form more useful for engineering applications. Irwin defined an energy release rate or the crack extension force tendency which can be determined from the stress and displacement fields in the vicinity of the crack tip rather then from an energy balance for elastic solid as a whole, as Griffith suggested.

Conservation integrals in elasticity have been widely applied to the fracture mechanics, among which the J integral is the most popular one. The J integral is path independent for elastic solids, and can be shown that the integral is identical to Irwin's energy release rate associated with the collinear extension of a crack in elastic solid, Rice[1].

The eXtended Finite Element Method (X-FEM) attempts to alleviate the computational challenges associated with mesh generation by not requiring the finite element mesh to conform to cracks, and in addition, provides a means to use higher-order elements or special finite elements without significant changes in the formulation. Building on prior work due to Belytchko et al.[2], foundations of the method were presented in Moes et al.[3] for 2-dimensional cracks.

The essence of the X-FEM lies in sub-dividing a model problem into two distinct parts: mesh generation for the geometric domain (cracks not included), and enriching the finite element approximation by additional functions that model the cracks and other geometric entities.

Modelling crack growth in a traditional finite element framework is cumbersome due to need for the mesh to match the geometry of the discontinuity. Many methods require remeshing of the domain at each time step. In the X-FEM the need for the remeshing is eliminated. The mesh does not change as the crack growths and is completely independent of the location and geometry of the crack. The discontinuities across the crack are modelled by enrichment functions.

2 EQUIVALENT DOMAIN INTEGRAL METHOD (J-EDI)

Rice[1] defined a path-independent J-integral for two-dimensional crack problems in linear and nonlinear elastic materials. As shown in the Fig. 1), J is the line integral surrounding a two-dimensional crack tip and is defined as

$$J_1 = \lim_{\Gamma_s \to 0} \int_{\Gamma_s} \left(W\delta_{1j} - \sigma_{ij} u_{i,1} \right) n_j d\Gamma \qquad i,j = 1,2 \tag{1}$$

where W is the strain energy density given by

$$W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} = \frac{1}{2}C_{ijkl}\varepsilon_{kl}\varepsilon_{ij} \tag{2}$$

and n_j is the outward normal vector to the contour integration Γ_S is around the crack tip (as shown in Fig. 1), σ_{ij} is stress tensor, ε_{ij} is strain tensor, C_{ijkl} is constitutive tensor and u_i are components vector of the displacement.

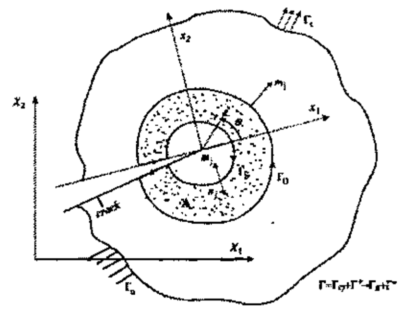


Figure 1: Conversion of the contour integral into an EDI

Knowles et al.[4] noted that this can be considered as the first component of a vector

$$J_k = \lim_{\Gamma_S \to 0} \int_{\Gamma_S} \left(W\delta_{kj} - \sigma_{ij} u_{i,k} \right) n_j d\Gamma \qquad i,j,k = 1,2 \tag{3}$$

which is also path independent.

Helen et al.[5] showed that

$$J = J_1 - iJ_2 = \frac{1}{E^*}\left(K_I^2 + K_{II}^2 + 2iK_I K_{II} \right) \tag{4}$$

where K_I and K_{II} are stress intensity factors for modes I and II respectively. Thus the values of energy release rates (J_1 and J_2) for crack extension perpendicular and parallel to the crack, respectively, will be given by:

$$J_1 = \frac{K_I^2 + K_{II}^2}{E^*} \qquad J_2 = \frac{-2K_I K_{II}}{E^*} \tag{5}$$

where

$$E^* = \begin{cases} E & \textit{plane strain} \\ \dfrac{E}{1-\nu^2} & \textit{plane stress} \end{cases} \tag{6}$$

Note that solution equation (5) is the intersection of circle and a hyperbola. Hence there exists more than one pair of stress intensity factors.

The contour integral (1) is not in a form best suited for finite element calculations. We therefore recast the contour integral into an equivalent domain form. The equivalent domain integral method (EDI) is an alternative way to obtain the J-integral. The contour integral is replaced by an integral over a finite-size domain. The EDI approach has the advantage that the effect of variable body forces can easily be included. The standard J-contour integral given by (3) is rewritten, by introducing a weight function $q(x_1, x_2)$ into the EDI. Hence, we define the following contour integral

$$\Psi = \int_{\Gamma} (W\delta_{kj} - \sigma_{ij} u_{i,k}) m_j q d\Gamma \qquad i,j,k = 1,2 \tag{7}$$

where is contour $\Gamma = \Gamma_0 + \Gamma^+ - \Gamma_S + \Gamma^-$ (Fig. 1), m_j is a unit vector outward normal to the corresponding contour (i.e. $m_j = n_j$ on Γ_0 and $m_j = -n_j$ on Γ_S), and q is a weight function defined as $q = 1$ inside the contour Γ and $q = 0$ for the domain outside Γ.

Taking the limit $\Gamma_S \to 0$, $q = 0$ on Γ_0 and the crack faces are assumed to be traction-free, the above equation becomes

$$J_k = -\lim_{\Gamma_S \to 0} \Psi = \lim_{\Gamma_S \to 0} \int_{\Gamma} (W\delta_{kj} - \sigma_{ij} u_{i,k}) m_j q d\Gamma \qquad i,j,k = 1,2 \tag{8}$$

Now applying the divergence theorem to (8), we obtain the following J-EDI

$$J_k = \int_A \left(\sigma_{ij} u_{i,k} - W\delta_{kj}\right) q_{,j} dA + \int_A \left(\sigma_{ij} u_{i,k} - W\delta_{kj}\right)_{,j} q dA \qquad i,j,k = 1,2 \tag{9}$$

where A is the area enclosed by Γ. Note that the second term in the above equation must vanish for linear-elastic materials[3,6] and that we have

$$J_k = \int_A \left(\sigma_{ij} u_{i,k} - W\delta_{kj}\right) q_{,j} dA \qquad i,j,k = 1,2 \tag{10}$$

In 3D case the J-EDI integral is converted into a volume integral[3,7] as

$$J_k = -\frac{1}{f} \int_V \left(W\delta_{kj} - \sigma_{ij} u_{i,k}\right) q_{,j} dV \qquad i,j,k = 1,3 \tag{11}$$

where $f = (2/3)\Delta$, with Δ being the thickness of the 3D element in the direction of the crack front.

3 NUMERICAL EVALUTATION OF THE J-INTEGRAL

The J-integral evaluation in the PAK program is based on the domain integration method described above. A direct evaluation of the contour integral is not practical in the finite element analysis (FEA) due difficulties in defining the integration path Γ. The conversion of the contour integral to the domain integral is exact for the linear elastic case and also for the nonlinear case if no unloading occurs[6].

When the material of the considered structure is homogeneous and the body forces are absent, the finite element implementation of (10) becomes very similar to that of the contour integral. The only difference is the introduction of the weight function q when (10) is used. With the isoparametric finite element formulation the distribution of q within the elements is determined by a standard interpolation scheme with use of the shape functions h_i:

$$q = \sum_{i=1}^{m} h_i Q_i \tag{12}$$

where Q_i are values of the weight function at the nodal points, and m is the number of nodes. The spatial derivatives of q can be found by use of the usual procedures for isoparametric elements.

The equivalent domain integral in 2D can be calculated as a sum of the discretized values of (10),[7]:

$$J_k = \sum_{\substack{elements \\ in\ A}} \sum_{p=1}^{P} \left[\left(\sigma_{ij} \frac{\partial u_i}{\partial X_k} - W\delta_{kj} \right) \frac{\partial q}{\partial X_j} \det\left(\frac{\partial X_m}{\partial \eta_n} \right) \right]_p w_p \qquad i,j,k,m,n = 1,2 \tag{13}$$

and the equivalent domain integral (11) in 3D is

$$J_k = \frac{1}{f} \sum_{\substack{elements \\ in\ V}} \sum_{p=1}^{P} \left[\left(\sigma_{ij} \frac{\partial u_i}{\partial X_k} - W\delta_{kj} \right) \frac{\partial q}{\partial X_j} \det\left(\frac{\partial X_m}{\partial \eta_n} \right) \right]_p w_p \qquad i,j,k,m,n = 1,3 \tag{14}$$

The terms within $[\cdot]_p$ are evaluated at the Gauss points with use of the Gauss weight factors for each point are w_p. The present formulation is for a structure of homogeneous material in which no body forces are present. For the numerical evaluation of the above integral, the domain A is set from the set of elements about the crack tip. The domain A is set contain all elements which have a node within a ball of radius r_d about the crack tip, Fig. 2). The function q is then easily interpolated within the elements using the nodal shape functions, according to (12), where $Q_i = 1$ for nodes within of domain A and $Q_i = 0$ for nodes out of A.

Since the FEM calculation of displacements, strains, stresses, etc., are based on the global coordinate system, the $(J_k)_{global}$ is evaluated first and then, if needed, transformed into $(J_k)_{local}$. The above expressions are represented by the local coordinates $x_k, (k = 1,2)$, which can be expressed in terms of the global coordinates X_i by the transformation:

$$x_i = \alpha_{ij}(\theta) X_j, \quad \alpha_{ij}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{15}$$

The same transformation also holds for the J_k integral[8]

$$\begin{Bmatrix} (J_1)_{local} \\ (J_2)_{local} \end{Bmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{Bmatrix} (J_1)_{global} \\ (J_2)_{global} \end{Bmatrix} \tag{16}$$

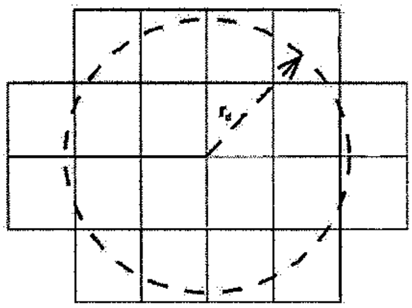


Figure 2: Domain integration for J-EDI

4 EXTENDED FINITE ELEMENT METHOD

In particular instance of 2-d crack modelling, the enriched displacement approximation is writen as[3]:

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I \in N_u} N_I(\mathbf{x}) \left(\mathbf{u}_I + \underbrace{H(\mathbf{x})\mathbf{a}_I}_{I \in N_a} + \underbrace{\sum_{\alpha=1}^{4} \Psi_\alpha(\mathbf{x})\mathbf{b}_I^\alpha}_{I \in N_b} \right) \tag{17}$$

where $\mathbf{u}_I$ is the nodal displacement vector associated with the continuous part of the finite element solution, $\mathbf{a}_I$ is the nodal enriched degree of freedom vector associated with the Heveisade (discontinuous) function $H(\mathbf{x})$, and $\mathbf{b}_I^\alpha$ is the nodal enriched degree of freedom vector associated with the elastic asymptotic crack-tip function $\Psi_\alpha(\mathbf{x})$. In the above equation N_u is the set of all nodes in the element; N_a is the set of nodes whose shape function support is cut by the crack; and N_b is the set of nodes whose shape function support is cut by the crack tip. With $\mathbf{x}$ we denote Descartes coordinates in 2d space.

The interior of the a crack is modelled by the generalized Heveisade enrichment function $H(\mathbf{x})$, where $H(\mathbf{x})$ takes on the value +1 above the crack and −1 below the crack:

$$H(\mathbf{x}) = \begin{cases} 1 & if (\mathbf{x}-\mathbf{x}^*)\cdot\mathbf{n} \geq 0 \\ -1 & if (\mathbf{x}-\mathbf{x}^*)\cdot\mathbf{n} < 0 \end{cases} \tag{18}$$

where $\mathbf{x}$ is a sample (Gauss) point, $\mathbf{x}^*$ (lies on the crack) is the closest point to $\mathbf{x}$, and $\mathbf{n}$ is unit outward normal to crack at $\mathbf{x}^*$.

The crack tip enrichment functions in isotropic elasticity are[3]:

$$\Psi(\mathbf{x}) = \{\Psi_1 \quad \Psi_2 \quad \Psi_3 \quad \Psi_4\} = \left\{\sqrt{r}\cos\frac{\theta}{2} \quad \sqrt{r}\sin\frac{\theta}{2} \quad \sqrt{r}\sin\frac{\theta}{2}\sin\theta \quad \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right\} \tag{19}$$

where r and θ denotes polar coordinates in the locale system at the crack tip.

5 THE FATIGUE LIFE ESTIMATION

The unstable crack propagation occurs when one of the stress intensity factors K_α $(\alpha = I, II, III)$ is equal or greater then experimentally determined material property K_C. The estimation of fatigue life can be updated for each crack extension. The crack growth equation provides a relation between the crack increment Δa and the increment in the number of load cycles ΔN. In case of cyclically loaded structures the number of load cycles equivalent to the crack increment can be determined by a numerical integration of the governing crack growth equation[9].

The Paris law is a simple but very often used model for description of the crack growth rate in the linear region under mode I. This law has the form

$$\frac{da}{dN} = C\Delta K^m \tag{20}$$

where ΔK is the stress intensity factor range, and C and m are the material constants. A shortcoming of the Paris law is that it neglects the influence of the peak stress and the threshold range.

The growth of cracks under mode I and mode II was first systematically studied by Iida et al.[10]. The results of their experiments showed that even a small ΔK_{II} increase would significantly increase the crack growth rate. However, they also observed that the crack tended to grow in the direction of minimum K_{II}. Some models take into account the mode II contribution. One way is by introducing an equivalent stress intensity factor ΔK_{Ieq} in the Paris equation

$$\frac{da}{dN} = C(\Delta K_{Ieq})^m \tag{21}$$

The maximum stress criterion can also be used to determine the equivalent mode I stress intensity factor, according to the following expression

$$K_{Ieq} = K_I \cos^3\frac{\theta_0}{2} - 3K_{II}\cos^2\frac{\theta_0}{2}\sin\frac{\theta_0}{2} \tag{22}$$

where θ_0 denotes the direction in which the crack is likely to propagate relative to the crack tip coordinate system, and ΔK_{Ieq} is found to be the K_{Ieq} range during one load cycle.

Tanaka[11] carried out experiments on cyclically loaded sheets of pure aluminum with initial cracks inclined to the tensile axis. As a by-product, the experiments formed the basis for a crack propagation law

$$\frac{da}{dN} = C\left(\Delta K_{eq}\right)^m \tag{23}$$

where

$$\Delta K_{eq} = (\Delta K_I^4 + 8\Delta K_{II}^4)^{1/4} \tag{24}$$

The above equation was developed on the assumptions that a) plastic deformation due to cyclic tension and transverse shear are not interactive, and b) the resulting displacement field is the sum of the displacements from the two modes.

For given crack geometry, by using of X-FEM one can defines the field of displacement and the stress state as well. Let $(\sigma^n, \mathbf{u}^n)$ denote stress state and displacement obtained in the n-th step of the simulation. Based on calculated stress and deformation states, which correspond to n-th step of simulation, by using of J-EDI method we can obtain values of the stress intensity factors K_I^n and K_{II}^n.

Obtained values of the stress intensity factor can be used for defining the angle of crack tip propagation θ_c^{n+1} and the increment of the crack growth Δa^{n+1} as well. With parameters $(\theta_c^{n+1}, \Delta a^{n+1})$ we can define new segment of crack, i.e., new crack geometry that will be initial configuration for n+1 simulation step.

Angle of propagation θ_c^{n+1} as well as increment of crack growth Δa^{n+1} can be defined in the local coordinate system associated to the n-th crack tip. Also, these parameters could be used for calculation of the coordinates of the new crack tip $\mathbf{x}_p^{n+1}$.

Angle of propagation θ_c^{n+1} could be calculated by criterion of maximum hoop stress[10]:

$$\theta_c^{n+1} = 2\tan^{-1}\left[\frac{-2K_{II}^n/K_I^n}{1+\sqrt{1+8(K_{II}^n/K_I^n)^2}}\right] \tag{25}$$

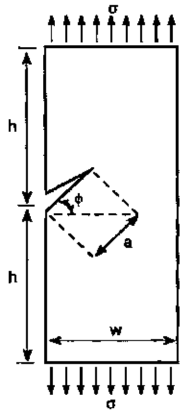
Crack growth increment Δa^{n+1} could be specified at the beginning of the solution procedure, in the term of percentage of the initial crack length, and unchanged kept during the rest of calculation. It is worth to notice that selection of the crack length increment value depends on initial crack length and numerical grid density as well. Also, with decreasing of the growth increment it is recommended to refining the grid of elements.

6 NUMERICAL EXAMPLES

In this section, we present several examples of calculation of stress intensity factors in case of crack under the assumption of plane strain and plane stress two-dimensional elasticity. We begin with a simple example of an edge crack to demonstrate the robustness of the above technique, and then present results for more complicated geometries. The results obtained with the PAK program will also be compared with results obtained by using the COSMOS program.

6.1 Plate with inclined crack edge

In this example we determine the stress intensity factor for both modes of fracture (opening K_I and shearing K_{II}) for a rectangular plate, with an inclined crack edge subjected to uniform uniaxial tensile pressure at the two ends.



$\sigma = 1\,psi$
$h = 2.5\,in$
$W = 2.5\,in$
$a = 1\,in$
$E = 30\times10^{6}\,psi$
$\nu = 0.3$
$\phi = 45^\circ$
$Thickness = 1\,in$

Figure 3: Plate with inclined crack edge

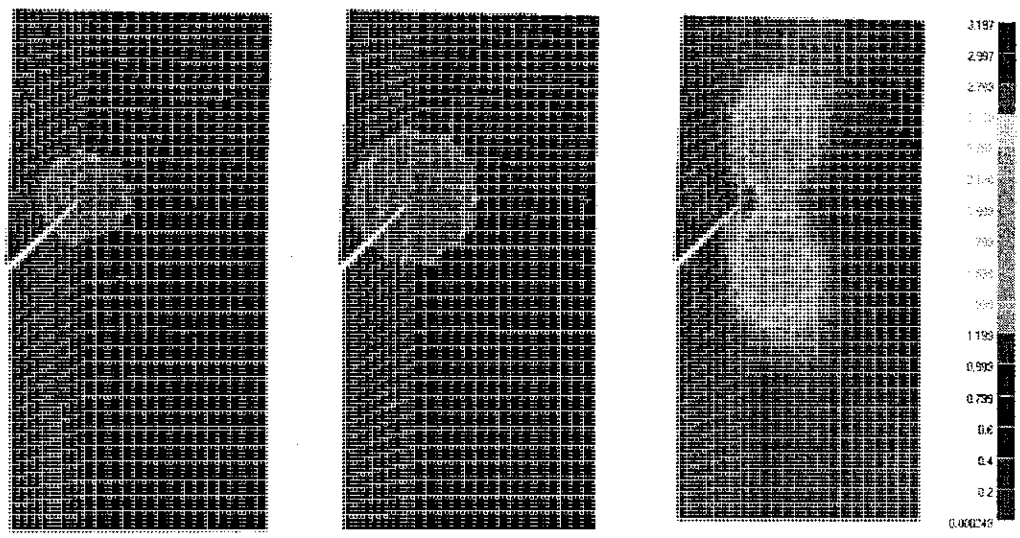


Figure 4: Domain integration for J-EDI and stress field

The full part has to be modeled since the model is not symmetric with respect to the crack. There is no restriction in our FE models, so that mesh can be either symmetric or non-symmetric with respect to the crack. Fig. 4) shows the first and second region of the integration for equivalent domain integral.

Results obtained by using J-EDI integral, incorporated in PAK software, are compared with results carried out with COSMOS J-contour integral and are shown in Table 1. Also, both sets of the numerical results are compared with the reference theoretical values.

		K_I *(N/A %)*	K_{II} *(N/A %)*
	Reference	1.85	0.88
8-node Element PAK	Path 1	1.877 *(1.4%)*	0.871 *(1.0%)*
	Path 2	1.907 *(3.0%)*	0.907 *(3.0%)*
8-node element COSMOS	Path 1	1.80 *(2.7%)*	0.872 *(0.9%)*
	Path 2	1.79 *(3.2%)*	0.874 *(0.6%)*

Table 1 : Comparison of results

In order to present robustness of the J-EDI procedure, that is built into the PAK software, the above example was used with different radii r_d of the integration domain and the results are shown in Table 2. Radius r_d was varied from $0.5\% a$ to $90\% a$, where a denotes crack length. It can be concluded from the Table 2 that the results are insensitive to the choice of the J-integral domain integration radius.

r_d *(% of a)*	*5*	*15*	*25*	*35*	*45*	*55*	*65*	*75*	*85*	*90*
K_I	*1.810*	*1.864*	*1.807*	*1.877*	*1.906*	*1.9075*	*1.9071*	*1.9089*	*1.929*	*1.931*
N/A(%)	*2.1*	*0.75*	*2.3*	*1.4*	*3.0*	*3.1*	*3.08*	*3.20*	*4.20*	*4.37*

Table 2 : Values of the factor K_I for different domain integration radius

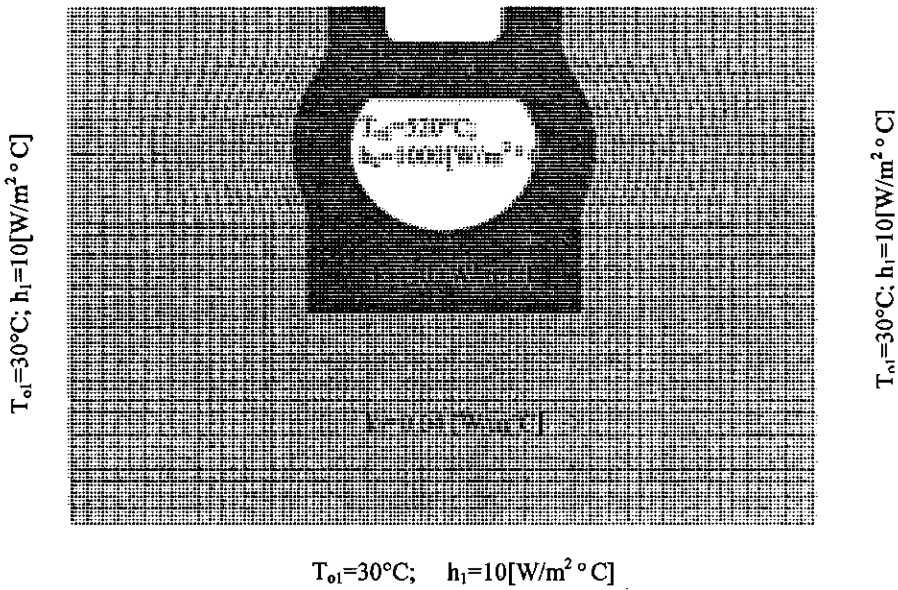
It can be seen from the results presented that the error *(N/A%)* is small, even with a unsymmetrical grid with respect to the crack.

6.2 Life assessment for steam turbine housing (2-D analysis)

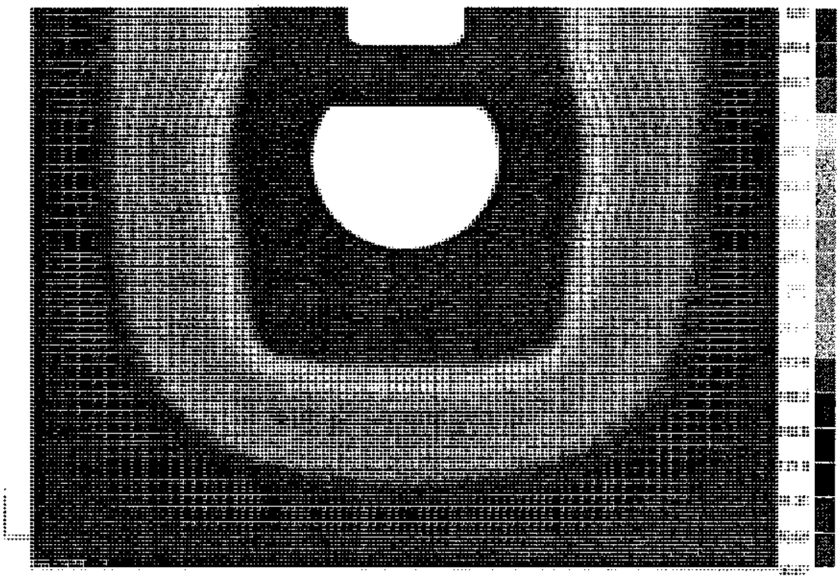
In this example[12,13], the stress intensity factor of the crack located in the steam turbine housing is calculated. After generating 2-D FE model of the lower housing part together with insulation, the following steps were carried out:

- Calculation of the temperature field in nominal regime as well as the corresponding stress field;
- Calculation of the stress and deformation fields of the turbine for different crack lengths (20–75 mm);
- Analysis of the influence of the crack length on the corresponding stress field as well as on the stress intensity factor;

For the purpose of calculation of the temperature field, 2-D grid consisted of 4400 8-nodes elements. Generated grid was comprised the space of the turbine housing and insulation.



a)



b)

Figure 5: a) 2D model for calculation of the temperature field ; b) Temperature field

In Fig. 5) the stress field induced by temperature and internal pressure is shown. The effective stress for 2-D turbine model without insulation, for the crack length 30 mm, is shown in Fig. 6).

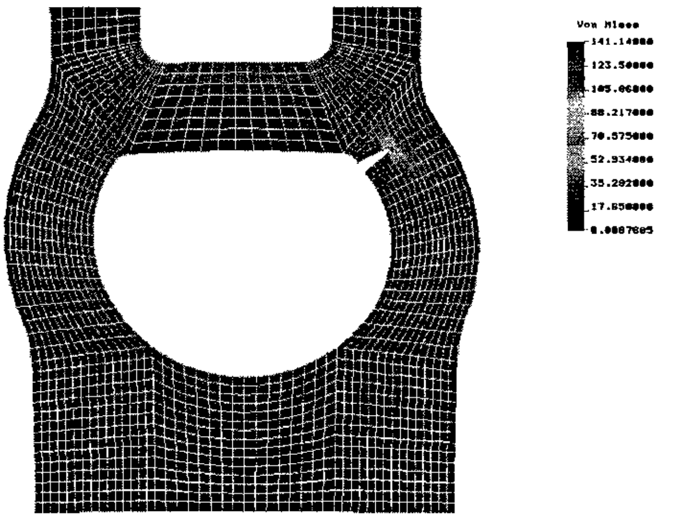


Figure 6: Effective stress field for crack length 30 mm

Fig. 7) shows the relationship between stress intensity factor K_I and crack length. It can be seen from Fig. 7) that by increasing the crack depth from 20 mm to 40 mm, the stress intensity factor increases for 30%. Also, with increasing of the crack length over 50 mm, the stress intensity factor increases more rapidly.

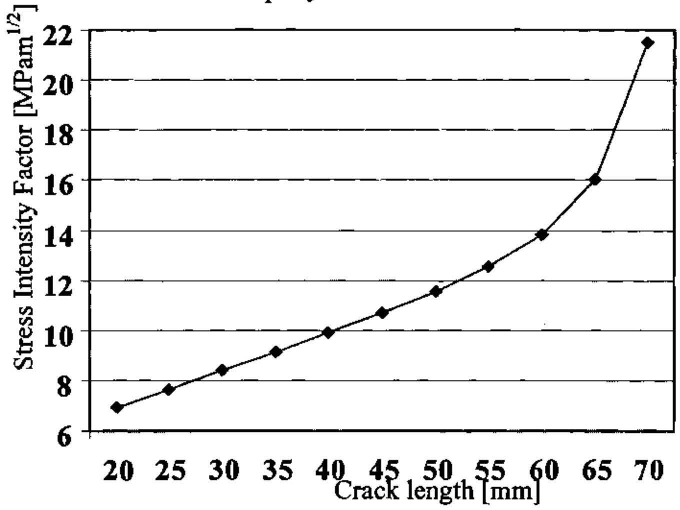


Figure 7: Relationship between stress intensity factor K_I and crack length

6.2 Life assessment for steam turbine housing (3-D analysis)

In this example[12,13] a 3-D analysis of the turbine housing is carried out. Using the original project documentation, 3-D geometrical model of the turbine is generated. In that 3-D object, the crack with different lengths (90 – 375 mm) and depth (20 – 40 mm) are assumed and modeled. The calculations are performed to investigate the influence of the crack length and crack depth on the value of maximum effective stress, as well as on the value of stress intensity factor. Lower part of the turbine housing has an axial plane of symmetry so that the 2-D model corresponds to the cross-section of that plane and the solid body of the housing.

For calculation of the temperature field, we used boundary conditions of thermal conduction according to Fig. 5). In order to reduce the number of elements in the 3-D grid, the critical quarter of the turbine is modeled. It is worth to emphasize that the cracks are located in that quarter as well as the steam intake with sharp edges that induce the stress concentration. In Fig. 8) shown the 3-D model. The calculated relationship between maximum effective stress and crack length for different crack depth is shown in Fig. 9).

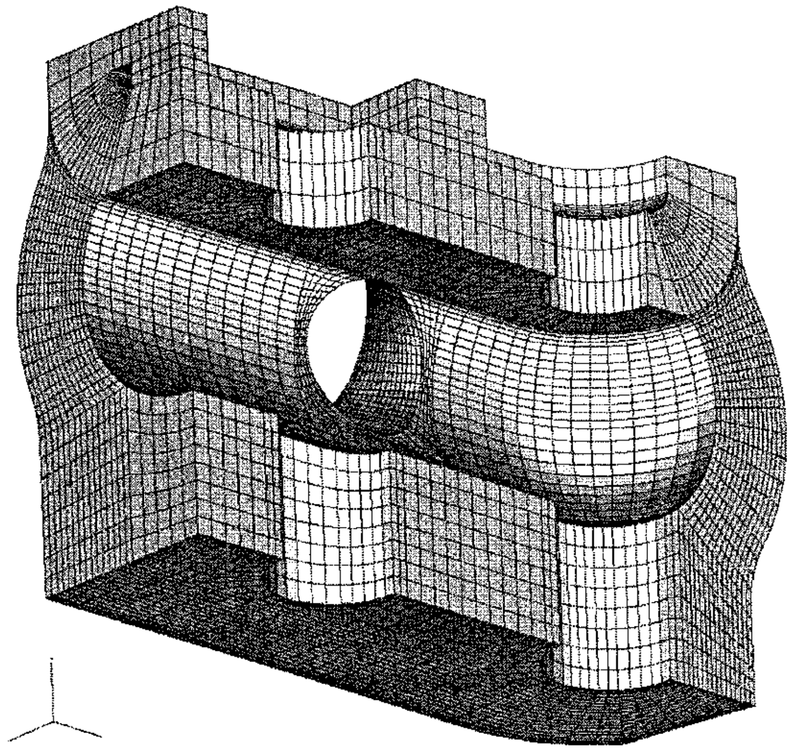


Figure 8: 3-D model

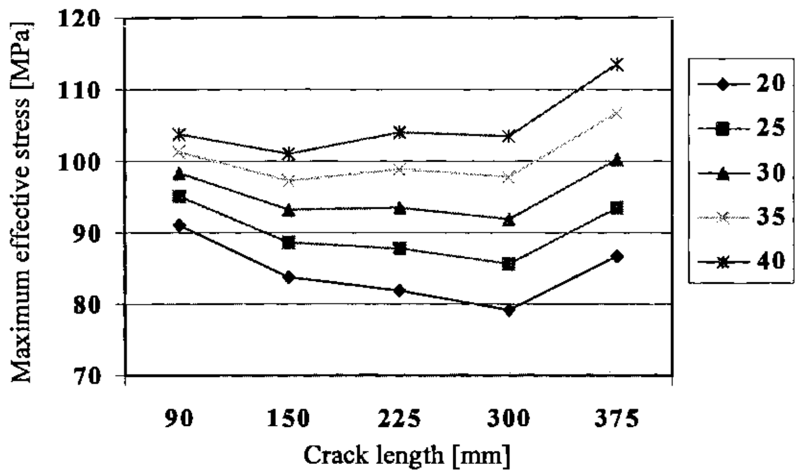


Figure 9: Relationship between maximum effective stress and crack length for different crack depth

It can be seen from Fig. 9) that variation in crack length from 90 mm to 375 mm, for the crack depth constant, has no significant influence on the effective stress. On the other hand, increase of the crack depth, for the crack length constant, leads to increase in the effective stress from 15 % to 30 %. Fig. 10) shows the field of the effective stress.

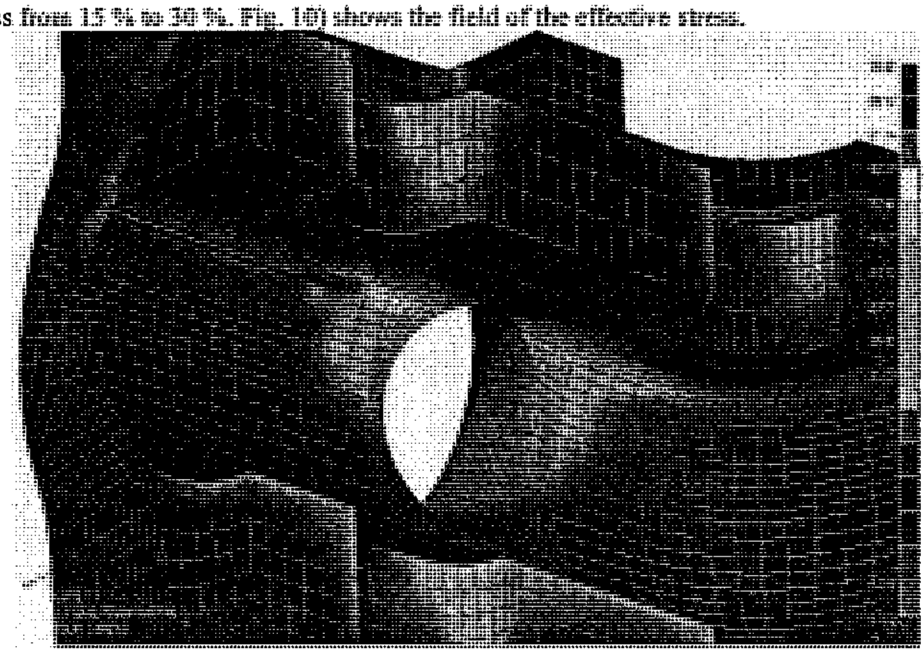


Figure 10: Field of the effective stress in vicinity of the crack (375x30 mm)

Dependence of the stress intensity factor on the crack length and crack depth is shown in Fig. 11). It is observed that increasing of the crack depth from 20 mm to 40 mm, for the crack length constant, leads to increase of the stress intensity factor from 15% to 30 %.

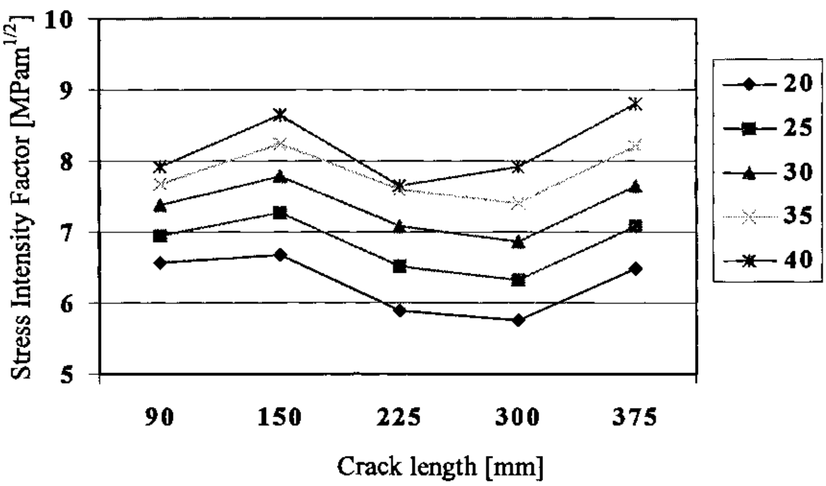


Figure 11: Relationship between stress intensity factor K_I and crack length for different crack depth

12 CONCLUSIONS

Based on the equivalent domain integral (EDI) method, very robust, efficient and reliable procedure for estimation of stress intensity factors is obtained. Application of the J-EDI integral is suitable for applications because it relies on use of the domain integrals rather than contour integrals.

In the X-FEM, the finite element method is enriched by adding special functions to the approximation using the notion of partition of unity. The crack was represented by H and NT functions. Discontinues function (H) was used to model the interior of the crack, and NT functions from the two-dimensional asymptotic displacement fields were used for the crack tip. These enrichment functions were added to the finite element approximation within the context displacement-based Galerkin formulation. A computational algorithm for crack growth using X-FEM was also presented.

Obtained numerical results show a small influence of the choice of the J-integral domain integration on value of the stress intensity factor. In addition to relatively simple test cases, the analysis of the complex 3-D problems is presented. The analysis shows that a stable crack growth is predicted in nominal regime of the analyzed structure, while 2-D analysis shows a rapid increase of the stress intensity factor for increasing the crack depth over 50 mm.

REFERENCES

[1] J. R. Rice, "A Path Independent Integral and Approximate Analysis of Strain Concentration by Notches and Cracks", *Journal of Applied Mechanics*, **35**, 379-386, (1968).

[2] T. Belytschko and T. Black, "Elastic crack growth in finite elements with minimal remeshing", *Int. J. Num. Meth. Engng*, **45**, 601-620, (1999).

[3] N. Moes, J. Dolbow, T. Belytschko, "A Finite Element Method for Crack Growth Without Remeshing", *Int. J. Num. Meth. Engng*, **46**, 131-150, (1999).

[4] J. Knowles and E. Sternberg, "On a class of conservation laws in linearised and finite elastostatics", *Arch. Rat. Mech. Anal.*, **44**, 187-211, (1972).

[5] T. Hellen and W. Blackburn, "The calculation of stress intensity factor for combined tensile and shear loading", *Int. J. Fract.*, **11**, 605-617, (1975).

[6] V. E. Sauma, *Fracture Mechanics*, Dept. of Civil Environmental and Architectural Engineering University of Colorado, Boulder Co 80309-0428, 2000.

[7] C.Y. Lin, *Determination of the Fracture Parameters in a Stiffened Composite Panel*, PhD Thesis, North Carolina State University, 2000.

[8] K. J. Ho and P. Glaucio, "T-stress, mixed-mode stress intensity factors, and crack initiation angles in functionally graded materials: a unified approach using the interaction integral method", *Comp. Method Appl. Mech. Engng*, **192**,1463-1494, (2003).

[9] A. M. Rye, *Fatigue Crack Initiation and Growth in Ship Structures*, PhD Thesis, Department of Naval Architecture and Offshore Engineering Technical University of Denmark, 1998.

[10] S. Iida and A. S. Kobayashi, "Crack-Propagation Rate in 7075-T6 Plates under Cyclic Tensile and Transverse Shear Loadings", *Journal of Basic Engineering*, 764-769, 1969.

[11] K. Tanaka, "Fatigue Crack Propagation from a Crack Inclined to the Cyclic Tensile Axis", *Engineering Fracture Mechanics*, **6**, 493-507, 1974.

[12] M. Živković, M. Kojić, R. Slavković, S. Vulović, V. Đorđevic, R.Vujanac, *Analysis of cracks in the steam turbine housing 4 in the thermal power plant Kolubara*, Report of Laboratory for engineering software, Faculty of Mechanical Engineering, University of Kragujevac, 2002.

[13] G. Jovičić, *Solving the fracture mechanics problems by extended finite element method*, PhD Thesis, Faculty of Mechanical Engineering, University of Kragujevac, 2005.

[14] M. Kojić, R. Slavković, M. Živković and N. Grujović, *PAK – Finite element program for structural analysis*, Faculty of Mechanical Engineering, University of Kragujevac.